PLEASURE TEST REVISION SERIES

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SUBJECT – MATHEMATICS FOR CLASS XII

Max. Marks: 100 **Time Allowed: 180 Minutes**

☞ ***General Instructions:***

i) Note that all questions are compulsory.

ii) Internal choices have been provided in some cases.

[SECTION – A]

Q01. For a non-singular matrix A, find $|adj(A^T)|$ if $A^{-1} = \begin{bmatrix} 1/5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Q02. Let f be a function defined as $f(x) = \dfrac{1}{2 - \sin 3x}$. Write the range of $f(x)$.

Q03. If $A = \begin{bmatrix} \cos Q & \sin Q \\ -\sin Q & \cos Q \end{bmatrix}$, find Q, $0 < Q < \dfrac{\pi}{2}$, where $A + A^T = I$.

Q04. Write the principal value of $\sin^{-1}(\sin 10)$.

Q05. Evaluate the integral: $\displaystyle\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$.

Q06. Check if the vectors $\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{c} = -8\hat{i} - \hat{j} + 3\hat{k}$ are coplanar vectors.

Q07. Find the distance between the planes $x + y + 2z = 1$ and $2x + 2y + 4z - 3 = 0$.

Q08. If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \dfrac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector then, find the angle between the vectors $\vec{a}$ and $\vec{b}$.

Q09. Evaluate $\displaystyle\int_0^{3/2} [x] dx$, where [.] is greatest integer function.

Q10. A matrix X has (*a* + *b*) rows and (*a* + 2) columns while the matrix Y has (*b* + 1) rows and (*a* + 3) columns. Both the matrices XY and YX exist. Find the values of *a* and *b*.

[SECTION – B]

Q11. Let $A = N \times N$ and * be a binary operation on A defined by $(a,b) * (c,d) = (a+c,\ b+d)$. Show that * is commutative and associative. Also, find the identity element for * on A, if any.

[OR] Let $f : [1,\infty) \to [1,\infty)$ be defined as $f(x) = 2^{x(x-1)}$ and is invertible. Find $f^{-1}(x)$.

Q12. If $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}\ldots\infty}}$ then, show that: $x\dfrac{dy}{dx} = \dfrac{y^2}{2 - y\log x}$.

Q13. Evaluate: $\displaystyle\int \frac{x^2}{x^4 - 3x^2 + 16} dx$.

Q14. Show that the normal at any point θ to the curves $x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

[OR] A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5m^3 per hour. Find the rate at which the level of water is rising at the instant when the depth of water in the tank is 4m.

Q15. Prove that: $2\tan^{-1}\left\{\tan\dfrac{\alpha}{2}\tan\left(\dfrac{\pi}{4} - \dfrac{\beta}{2}\right)\right\} = \tan^{-1}\left(\dfrac{\sin\alpha.\cos\beta}{\cos\alpha + \sin\beta}\right)$.

Q16. Discuss the continuity and differentiability of $f(x) = x - 3|\ \forall x \in R$ at $x = 3$. What do you conclude from the observation?

Q17. Find equation of a plane passing through (1,2,1) and perpendicular to the line joining the points (1,4,2) and (2,3,5). Also, find the perpendicular distance of the plane from origin.

Q18. Evaluate: $\displaystyle\int_{-2}^{2} \frac{dx}{1 + |x - 1|}$. **[OR]** Evaluate: $\displaystyle\int_{\pi/3}^{\pi/2} \frac{\sqrt{1 + \cos x}}{(1 - \cos x)^{3/2}} dx$.

Q19. If $\vec{a}$ and $\vec{b}$ are two vectors, then show that: $\left(\vec{a} \times \vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$.

Q20. **A)** Which equation of curve would satisfy $\frac{dy}{dx}=\sin(10x+6y)$ such that it passes through origin?

B) Write the order and degree of the differential equation: $\left(\frac{d^2y}{dx^2}\right)^5+\frac{4\left(\frac{d^2y}{dx^2}\right)^3}{\left(\frac{d^3y}{dx^3}\right)}+\left(\frac{d^3y}{dx^3}\right)=x^2-1$.

Q21. A die is thrown again and again until the number 6 is obtained three times. Find the probability that the third six comes in the seventh toss. "Outdoor games should be preferred over indoor games." Why?

[OR] Anshu and Manshu throw a die alternatively till one of them gets a 'six' and wins the game. Find their respective probabilities of winning, if Manshu starts the game. "Outdoor games should be preferred over indoor games." Why?

Q22. If $\mathrm{A}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ then, prove that $\left(a\mathrm{I}+b\mathrm{A}\right)^n=a^n\mathrm{I}+na^{n-1}b\mathrm{A}$, $n\in\mathrm{N}$ and I is the identity matrix of order 2.

[SECTION – C]

Q23. If $\mathrm{A}=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. Hence use it to solve the following system of equations:

$2x-3y+5z=16,\ 3x+2y+4=4z,\ x+y-2z=-3.$

[OR] If it is known that a, b, c are real numbers, and it is known that $\begin{vmatrix}b+c & c+a & a+b\\ c+a & a+b & b+c\\ a+b & b+c & c+a\end{vmatrix}=0$ then, show that either $a+b+c=0$ or, $a=b=c$.

Q24. A point on the hypotenuse of a right angled triangle is at the distances *a* and *b* from the sides of the triangle. Show that the minimum length of hypotenuse is $(a^{2/3}+b^{2/3})^{3/2}$.

Q25. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10, 1/5, 1/10 and 2/5. The probabilities that he will be late are 1/4, 1/3, and 1/12, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train? 'Public transport should be encouraged.' Why?

Q26. Find the area of the region: $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$.

Q27. Find the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})=0$ and $\vec{r}.(3\hat{i}+2\hat{j}+\hat{k})=0$. Show that this line is equally inclined to $\hat{i}$ and $\hat{k}$ and makes an angle $\frac{1}{2}\sec^{-1}(3)$ with $\hat{j}$.

Q28. A farmer mixes two brands P and Q of cattle feed. Brand P, costing ₹250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing ₹200 per bag, contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag? "Animals also require balanced diet for their growth." Do you agree? Why or, why not?

Q29. Evaluate: $\int\frac{\sin^{3/2}x+\cos^{3/2}x}{\sqrt{\sin^3x\cos^3x\sin(x+a)}}dx$.

[OR] Evaluate: $\int_0^1\tan^{-1}(1-x+x^2)\,dx$.

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